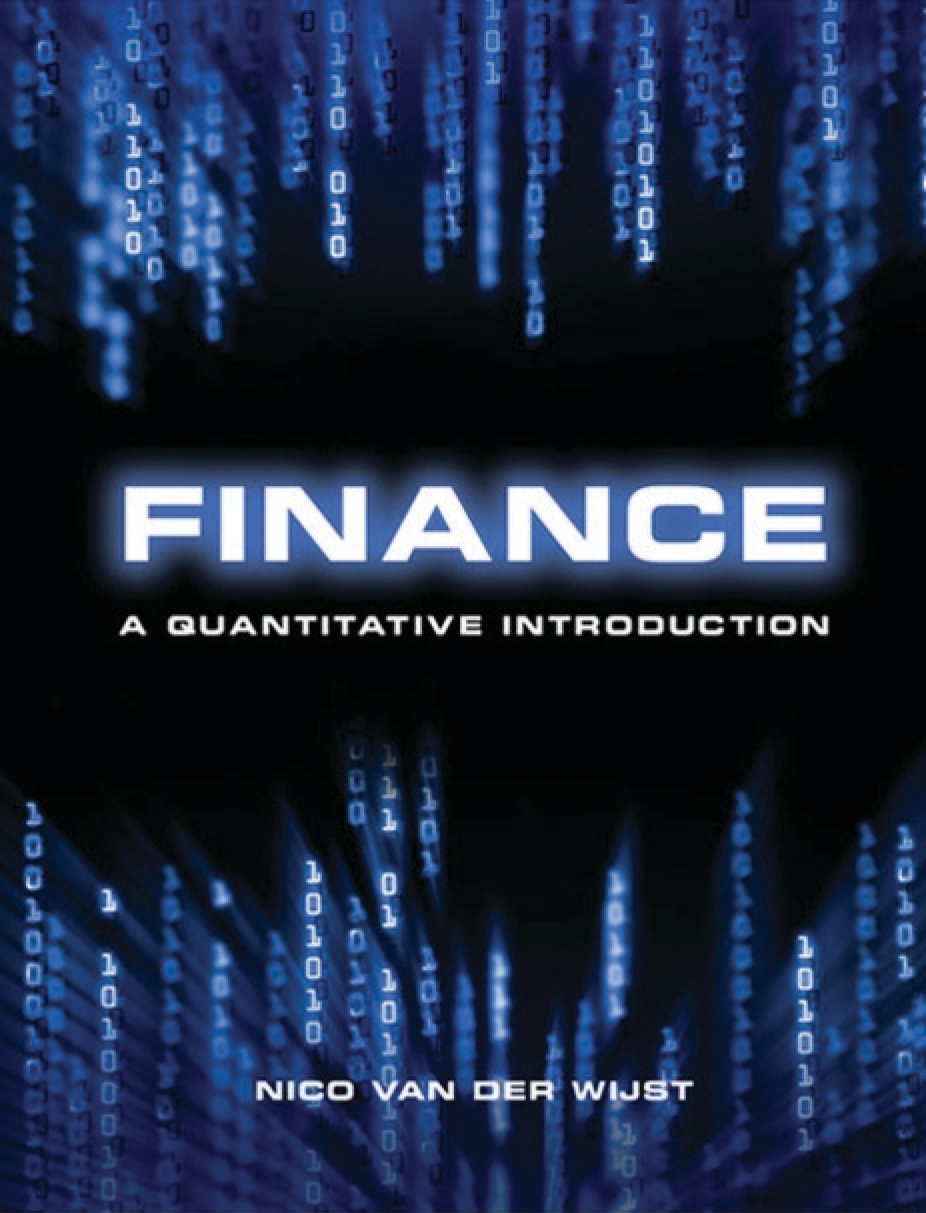

FINANCE

A QUANTITATIVE INTRODUCTION

NICO VAN DER WIJST

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Finance

By providing a solid theoretical basis, this book introduces modern finance to readers, including students in science and technology, who already have a good foundation in quantitative skills. It combines the classical, decision-oriented approach and the traditional organization of corporate finance books with a quantitative approach that is particularly well suited to students with backgrounds in engineering and the natural sciences. This combination makes finance much more transparent and accessible than the definition-theorem-proof pattern that is common in mathematics and financial economics. The book's main emphasis is on investments in real assets and the real options attached to them, but it also includes extensive discussion of topics such as portfolio theory, market efficiency, capital structure and derivatives pricing. *Finance: A Quantitative Introduction* equips readers as future managers with the financial literacy necessary either to evaluate investment projects themselves or to engage critically with the analysis of financial managers.

A range of supplementary teaching and learning materials are available online at www.cambridge.org/wijst.

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Finance

A Quantitative Introduction

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Acronyms

APT	Arbitrage Pricing Theory
APV	adjusted present value
BIS	Bank for International Settlements
caar	cumulative average abnormal return
CAPM	Capital Asset Pricing Model
CEO	chief executive officer
CFO	chief financial officer
CML	capital market line
DCF	discounted cash flow
EMH	Efficient Market Hypothesis
FV	future value
IPO	initial public offering
IRR	internal rate of return
Nasdaq	National Association of Securities Dealers Automated Quotations
NPV	net present value
NYSE	New York Stock Exchange
OCC	opportunity cost of capital
OECD	Organisation for Economic Cooperation and Development
OTC	over the counter
PV	present value
S&P	Standard & Poor's
SDE	stochastic differential equation
SEC	Securities and Exchange Commission
SML	security market line
VaR	value at risk
WACC	weighted average cost of capital

Preface

Finance has undergone spectacular changes in the last four decades, both as a profession and as a scientific discipline. Before 1973 there were no option exchanges and there was no generally accepted model to price options. Today, the worldwide trade in derivative securities represents a much larger money amount than the global production of goods and services. The famous Black and Scholes option-pricing formula and its descendants are used in financial markets all over the world where an enormous number of derivative securities are traded every day. Professionals in sectors like engineering, telecommunications and manufacturing regularly find that their projects are evaluated with techniques such as real options analysis. Understanding the basic concepts of finance is increasingly becoming a prerequisite for the modern work place.

Many scientific developments in finance are fuelled by the use of quantitative methods; finance draws heavily on mathematics and statistics. This gives students and professionals who are familiar with quantitative techniques an advantage in mastering the principles of finance. As the title suggests, this book gives an introduction to finance in a manner and ‘language’ that are attuned to an audience with quantitative skills. It uses mathematical notations and derivations where appropriate and useful. But the book’s main orientation is conceptual rather than mathematical; it explains core financial concepts without formally proving them. Avoiding the definition-theorem-proof pattern that is common in mathematical finance allows the book to use the more natural order of first presenting an insight from financial economics, then demonstrating its empirical relevance and practical applicability, and concluding with a discussion of the necessary assumptions. This ‘reversed order’ reduces the scientific rigour but it greatly enhances the readability for novice students of finance. It also allows the more demanding parts to be skipped or made non-mandatory without loss of coherence.

The need for a book like this arose during the many years that I have been teaching finance to science and technology students. Their introductory years give these students a good working knowledge of quantitative techniques, so they are particularly well placed to study modern finance. However, almost all introductory textbooks in finance are written for MBA students, who have a much less quantitative background. In my experience, teaching finance to numerate students using an MBA textbook is an unfortunate combination. It forces the teacher to supply much additional material to allow students to use their analytical skills and to highlight the quantitative aspects that are severely understated in MBA textbooks. Of course, there are many textbooks in finance that are analytically more advanced, but these are usually written for a second or third course. They assume familiarity with the terminology and basic concepts of finance, which first-time readers do not possess. This is also the case for introductory textbooks in financial economics, or the ‘theory of finance’. In addition, many of these books are written in the definition-theorem-proof pattern, which makes them, in my opinion, less suitable for introductory courses. Students’ first meeting with finance should be an appetizer that arouses their interest in finance as a science, shows them alternative uses for the quantitative techniques they have acquired, and welcomes them to the wonderful world of financial modelling. Formal proofs are not instrumental in that.

Readership

This book is primarily written for science and technology students who include a course in finance or project valuation in their study programmes. Most study programmes in mathematics, engineering, computer science and the natural sciences offer the opportunity to include such elective subjects; their typical place is late in the bachelor programme or early in the master programme. The book can be used as the only text for a course in finance or as one of several if other management aspects are included, such as project planning and organization. Given the limited room for these courses in most study programmes the book has to be concise, but it takes students from discounting to the Black and Scholes formula and its applications. To limit its size, the main emphasis is on investments in real assets and the real options attached to them. This is the area of finance that prospective natural scientists and engineers are most likely to meet later in their careers. Of course, a thorough analysis of such investments requires a theoretical basis in finance that includes portfolio theory and the pricing models based on it, market efficiency, capital structure, and derivatives pricing. Topics with a less direct connection with real assets are omitted, such as bond pricing, interest rate models, market microstructure, exotic options, cash and receivables management, etc.

I have also used the material in this book for intermediate courses in finance for business school students. The purpose of these courses is to deepen students’ theoretical understanding of finance and to prepare them for more specialized subjects in, for example, continuous-time finance and derivatives pricing. The step from an introductory MBA book to a specialized text is often too large, and this book can fruitfully be used to bridge the gap. It introduces students to techniques that they will meet in later courses, but in a much more accessible and less formal way than is usual in the specialized literature. Greater accessibility is increasingly required because of the growing diversity in business school students’ backgrounds. In my experience, students find the material in the book both interesting and demanding, but most students rise to the challenge and successfully complete the course.

A final use that I have made of the book’s material is for a permanent education course aimed at professionals in science and technology and technical project leaders. After some years of work experience, many professionals feel the need for more knowledge about the way financial managers decide about projects, particularly how they value the flexibility in projects with real options analysis. The scope and depth of the book are sufficient to make such professionals competent discussion partners of financial managers in matters of project valuation, including the aspects of strategic value.

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Nico van der Wijst
Kräftriket, Stockholm, 2013

1 Introduction

This opening chapter introduces finance as a scientific discipline and outlines its main research tools. We also take a brief look at the book's central theme, the calculation of value, and the main ways to account for risk in these calculations. To illustrate the differences between finance and the natural sciences we compare results of a Nobel prize-winning financial model with measurements of a NASA space probe.

1.1 Finance as a science

1.1.1 What is finance?

Finance studies how people choose between uncertain future values. Finance is part of economics, the social science that investigates how people allocate scarce resources, that have alternative uses, among competing goals. Both scarcity, i.e. insufficient resources to achieve all goals, and possible alternative uses are necessary ingredients of economic problems. Finance studies such problems for alternatives that involve money, risk and time. Financial problems can refer to businesses, in which case we speak of corporate finance, but also to individuals (personal finance), to governments (public finance) and other organizations. Financial choices can be made directly or through agents, such as business managers acting on behalf of stockholders or funds managers acting on behalf of investors. For the most part, we shall study choices made by businesses in financial markets, but the results have a wider validity. As we shall see, financial markets facilitate, simplify and increase the possibilities to choose. Some typical problems we will look at are:

- Should company X invest in project A or not?
- How should we combine stocks and risk-free borrowing or lending in our investment portfolio?
- What is the best way to finance project C?
- How can we price or eliminate (hedge) certain risks?
- What is the value of flexibility in investment projects?

Finance as a scientific discipline (also called the 'theory of finance' or 'financial economics') seeks to answer such questions in a way that generates knowledge of general validity. It evolved from the descriptive science it was about 100 years ago into the analytic science it is now. Modern finance draws heavily on mathematics, statistics and other disciplines, and many scientists working in finance today started their careers in the natural sciences. Table 1.1 lists some milestones in the development of finance over the past century as well as some of the people whose work we shall meet. The importance of their

Table 1.1 *Milestones in the development of finance*

Name	Period	Nobel prize	Topics
J. Fisher	1930s	–	Optimal investment/consumption
K. Arrow	1950s	1972	State-preference theory
G. Debreu	1950s	1983	State-preference theory
J. Nash	1950s	1994	Game theory
H. Markowitz	1950s	1990	Portfolio theory
F. Modigliani	1950–60s	1985	Capital structure, cost of capital
M. Miller	1950–60s	1990	Capital structure, cost of capital
P. Samuelson	1960s	1970	Market efficiency
W. Sharpe	1960s	1990	Capital Asset Pricing Model
R. Merton	1970s	1997	Option pricing
M. Scholes	1970s	1997	Option pricing
F. Black	1970s	–	Option pricing

contributions is reflected in the Nobel prizes awarded to them: we will be standing on the shoulders of these giants.

Finance is also a tool box for solving decision problems in practice; this part is usually referred to as managerial finance. There is not always a direct relation between practical decision making and scientific results. For a number of practical problems there is no scientifically satisfactory solution. Conversely, some scientific results are still far from practical applications. But generally the insights from the theory of finance are also applied in practice and usually well beyond the strictly defined context they were derived in. Modern portfolio theory, Black and Scholes' option-pricing formula, risk-adjusted discount rates and many more results all have found their way into practice and are now applied on a daily basis.

1.1.2 How does finance work?

As in many other sciences, the main tools in finance are the mathematical formulation (i.e. modelling) of theories and their empirical testing. What makes finance special among social sciences is that financial markets lend themselves very well to modelling and testing, as well as application of the results. The list of Nobel prizes in Table 1.1 is testimony to successful applications of these tools in finance.

Scientific research in finance usually has an actual problem as its starting point. The problem is first made manageable by making simplifying assumptions with regard to, for example, investor behaviour and the financial environment investors operate in. The stylized problem is then translated into mathematical terms (modelled) and the analytical power of mathematics is used to formulate predictions in terms of prices or hypotheses. The predictions are tested by confronting them with real-life data, such as prices in financial markets, or accounting and other data. If the tests do not reject the theories we can apply their results to practical decisions, such as buying or selling in a financial market, accepting or rejecting an investment proposal, or choosing a capital structure for a project or a company. Alternatively, we can use the test results to adapt the theory. This gives a full cycle of scientific research, from formal theories to tests and practical applications. Figure 1.1 illustrates the interlocking cycles of scientific and applied research.

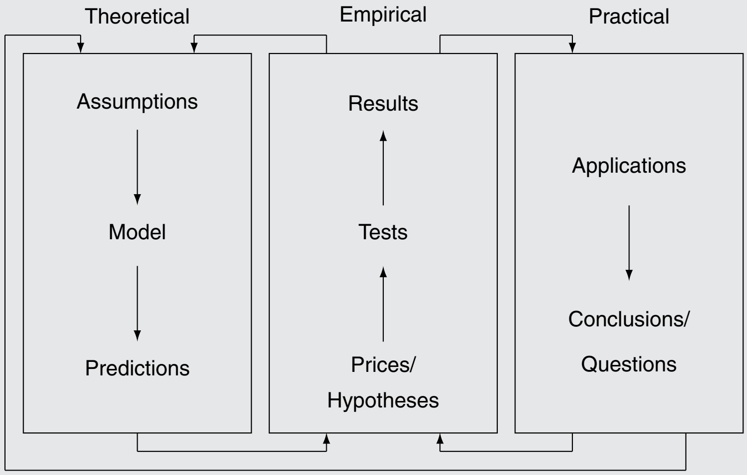

Figure 1.1 The interlocking cycles of scientific and applied research

Option pricing is a good example to illustrate the workings of finance. For many years, finding a good model to price options was an actual and very relevant problem in finance. Black and Scholes cracked this puzzle by making the simplifying assumptions of greedy[1] investors, a constant interest rate and stock price volatility and frictionless markets (we shall look at all these concepts later on). They then translated the problem in mathematical terms by formulating stock price changes as a stochastic differential equation and the option's payoff at maturity as a boundary condition. The analytical power of mathematics was used to solve this 'boundary value problem' and the result is the famous Black and Scholes option-pricing formula. Empirical tests have shown that this formula gives good predictions of actual market prices. So we can use the model to calculate the price of a new option that we want to create and sell ('write' an option) or to hedge (i.e. neutralize) the obligations from another contract, e.g. if we have to deliver a stock in three months' time. In fact, thousands of traders and investors use this formula every day to value stock options in markets throughout the world.

Scientific research does not necessarily begin with a problem and assumptions, it can also start in other parts of the cycles in Figure 1.1. For instance, in the 1950s statisticians analyzed stock prices in the expectation of finding regular cycles in them, comparable to the pig cycles in certain commodities.[2] All they could find were random changes. These empirical results later gave rise to the Efficient Market Hypothesis, which was accurately and succinctly worded by Samuelson as ‘properly anticipated prices fluctuate randomly’. Similarly, Myers’ Pecking Order Theory of capital structure is based on the observation that managers prefer internal financing to external, and debt to equity.

---

[1] This is not a moral judgement but the simple assumption that investors prefer more to less, mathematically expressed in the operator max[.].

[2] Pig cycles are periodic fluctuations in price caused by delayed reactions in supply, named after cycles in pork prices corresponding to the time it takes to breed pigs.

1.2 A central issue

A central issue in finance is the valuation of *assets* such as investment projects, firms, stocks, options and other contracts. In finance, the value of an asset is not what you paid for it when you bought it, nor the amount the bookkeeper has written somewhere in the books. It is, generally, the present value of the cash flows the asset is expected to generate in the future or, in plain English, what the expected future cash flows are worth today. That value depends on how risky those cash flows are and how far in the future they will be generated. This means that value has a *time* and an *uncertainty* dimension; the pattern in time and the riskiness both determine the value of cash flows.

As we shall see in the next chapter, the time value of money is expressed in the risk-free interest rate. That rate is used to ‘move’ riskless cash flows in time: discount future cash flows to the present and compound present cash flows to the future. Since the rate is accumulated (compounded) over periods, the future value of a cash flow now increases with time. Similarly, cash flows further in the future are ‘discounted’ more and thus have a lower present value. We can express this a bit more formally in a general present-value formula (where t stands for time):

$$Value = \sum_t \frac{Exp\left[Cash\,flows_t\right]}{(1 + discount\ rate_t)^t} \tag{1.1}$$

The numerator of the right-hand side of (1.1) contains the expected cash flow in each period. If the cash flow is riskless, the future amount is always the same, no matter what happens. Such cash flows can be discounted at the risk-free interest rate. If the cash flow is risky, the future amount can be higher or lower, depending on the state of the economy, for example, or on how well a business is doing. The size of a risky cash flow has to be expressed in a probabilistic manner, for example 100 or 200 with equal probabilities. The expectation then is the probability weighted average of the possible amounts: $\sum_i p_i CFL_i$ where p_i is the probability and CFL_i the cash flow. In the example, the expected cash flow is $0.5 \times 100 + 0.5 \times 200 = 150$.

There are three different ways to account for risk in the valuation procedure. The first way is to adjust the discount rate to *a risk-adjusted discount rate* that reflects not only the time value of money but also the riskiness of the cash flows. For this adjustment we can use a beautiful theory of asset pricing, called the Capital Asset Pricing Model (CAPM) or, alternatively, the equally elegant and more general but less precise Arbitrage Pricing Theory (APT). The second way is to adjust the risky cash flows so that they become certain cash flows that have the same value as the risky ones. These *certainty equivalent cash flows* can be calculated with the CAPM or with derivative securities such as futures, and they are discounted to the present at the risk-free interest rate. The third way is to redefine the *probabilities*, that are incorporated in the expectations operator, in such a way that they contain pricing information. Risk is then ‘embedded’ in the probabilities and the expectation calculated with them can be discounted at the risk-free interest rate. Changing probabilities is the essence of the Black–Scholes–Merton Option Pricing Theory, and it accounts for risk in a fundamentally different way than the CAPM or APT. We shall look at all three methods in detail and use them to analyze questions and topics such as these:

- What risks are there? Are all risks equally bad? Is risk always bad?
  We will see that some risks don't count and that risk can even be beneficial to some investments and people.
- Portfolio theory and valuation models
  They demonstrate why investments should not be evaluated alone, but combined.
- Market efficiency
  that explains why you, and your pension fund, cannot quickly get rich if markets function properly.
- The variety of financial instruments
  and how they helped to create the credit crunch.
- Capital structure
  or why some projects are easy to finance and others are not.
- The wild beasts of finance: options and other derivatives
  and why most projects and firms are options.
- Real options analysis
  and how flexibility can make unprofitable projects profitable.
- Modern contracting and incentive theory
  which explains why good projects can be turned down and bad projects can be accepted.

1.3 Difference with the natural sciences

The natural sciences generally study phenomena that, at least in principle, can be very precisely measured. Moreover, the relations between different phenomena can often be accurately predicted from the laws of nature and/or established in experimental settings that control all conditions. As a result, observations in natural sciences such as physics and chemistry usually show little dispersion around their theoretically predicted values.

Finance, however, is a social science: it studies human behaviour. Controlled experiments are practically always impossible. Financial economists cannot keep firms in an isolated experiment, control all economic variables and then measure how firms react to changes in the interest rate that the experimenter introduces, for instance. They can only observe firms in some periods with low interest rates and other periods with high interest rates. But it is not only the interest rate that changes from period to period; everything else changes as well. Hence, financial data consist of noisy, real-life observations and not clean, experimental data. Furthermore, it is impossible to control for all other factors in the statistical analyses that are used to estimate financial relations. So these relations are necessarily incomplete. As a result, observations in finance usually are widely dispersed around their theoretically predicted values. Science and technology students may need some time to acquaint themselves with the nature of financial relations. An extreme example from both sciences will illustrate the differences.

Figure 1.2 plots data collected by NASA's Wilkinson Microwave Anisotropy Probe (WMAP), a satellite that has mapped the cosmic microwave background radiation. That is the oldest light in the universe, released approximately 380,000 years after the birth of the universe 13.73 billion years ago. WMAP produced a fine-resolution full-sky map of the microwave radiation using differences in temperature measured from opposite directions. These differences are minute: one spot of the sky may have a temperature of 2.7251° Kelvin, another spot 2.7249° Kelvin. It took a probe of \$150 million to measure them. Figure 1.2 shows the relative brightness (temperature) of the spots in the map versus the size of the spots (angle). The shape of the curve contains a wealth of information about the history of the universe (see NASA's website at http://map.gsfc.nasa.gov/). The point here is that the observations, even of the oldest light in the universe, show very little dispersion.

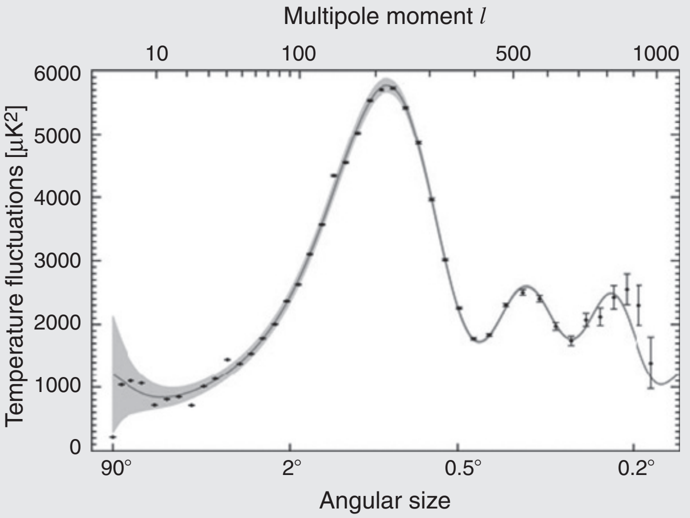

Figure 1.2 The angular spectrum of the fluctuations in the WMAP full-sky map. Credit: NASA/WMAP Science Team

Figure 1.3 plots the return versus the risk of companies in the Nasdaq-100 index in the period 4 October 2010 to 30 September 2011. As the name suggests, this index includes 100 of the largest US and international non-financial securities listed on Nasdaq, the world's first completely electronic stock market. Giants such as Apple, Adobe, Dell, Google, Intel and Microsoft are included in the data. Return is measured as the percentage price return (changes in stock price over the year, adjusted for dividends). Risk is the company's beta coefficient, which measures the contribution of the company's stock to the variance of a well-diversified portfolio.[3] The straight line is a theoretical model, the CAPM, for which Sharpe was awarded the Nobel prize in 1990. It gives the expected return of asset i, r_i, as a function of its beta coefficient β_i, the risk-free interest rate r_f

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[3] The beta coefficients are calculated relative to the Nasdaq-100 index using daily returns (adjusted for events such as dividends and splits) from 4 October 2010 to 30 September 2011.

and the expected return on the market portfolio r_m:

$$E(r_i) = r_f + (E(r_m) - r_f)\beta_i$$

In the period October 2010 to September 2011 the risk-free interest rate in the USA was close to zero, say 0.5 per cent, and the return of the Nasdaq-100 index was 9.5 per cent, so the relation is:

$$r_i = 0.5 + (9.5 - 0.5)\beta$$

which is the formula of the plotted line. All these concepts are discussed in detail later on. The point here is that the observations, even of the largest US companies, show a very large dispersion, even around a Nobel prize-winning theoretical model.

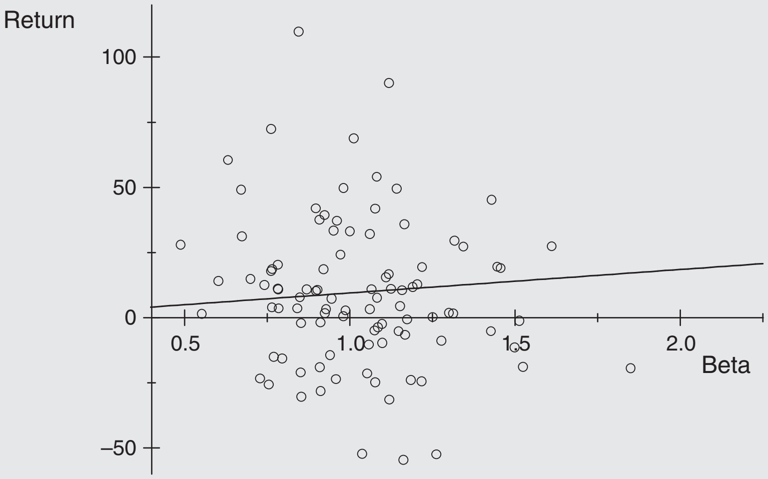

Figure 1.3 Risk–return relationship for Nasdaq-100 companies, October 2010 to September 2011

Of course, this extreme example does not imply that financial economics cannot predict at all, nor that empirical relations cannot explain more than a few per cent of variations in, for example, stock prices. On the contrary, the Fama–French three-factor model, which we shall meet in Chapter 3, explained more than 90 per cent of the variance in stock returns when it was first estimated. But it illustrates that empirical relations in finance have a different character compared with those in the natural sciences.

There is an additional reason why financial relations are less precise. Financial economics studies how people choose between uncertain future values. These future values are to a very large extent unpredictable, not because financial economists are not good at their jobs but because properly functioning financial markets make them unpredictable. In such markets, accurate predictions ‘self-destruct’. For example, if news becomes available from which investors can reliably predict that the value of a stock will double over the next month, they will immediately buy the stock and keep on buying it until the doubling is included in the price. So the ‘surprise’ is instantly incorporated into the price and the price changes over the rest of the month depend on new news, which is unpredictable by definition. Unlike most other sciences, financial economics has developed a coherent theory, the Efficient Market Theory, that explains why some of its most important study-objects, such as stock price changes, should be unpredictable.

The same information effects that make future values unpredictable also make observed, historical data noisy, as Figure 1.3 shows. Returns (or stock price changes) can be regarded as the sum of an unobserved expected part, plus an unexpected part that is caused by the arrival of new information. There is a constant flow of economic news from all over the world and much of it is relevant for stock prices. As a result, the unexpected part is large relative to the expected part. For example, daily stock price changes are typically in the range from -2 per cent to $+2$ per cent. If a stock is expected to have an annual return of 20 per cent and a year has 250 trading days, the daily expected return is $20/250 \approx 0.08$ per cent, very small compared with the observed values. Any model for expected stock returns will therefore have a high residual variance, i.e. explain only a small proportion of the variance of observed stock returns.

Finally, because efficient markets make price changes unpredictable, a high residual variance is a positive, not a negative, quality indicator of financial markets. The better financial markets function, the more unpredictable they are. To illustrate this, consider the results of studies which have shown that residual variance has increased over time (Campbell *et al.*, 2001), that it *increases* with the sophistication of financial markets (Morck *et al.*, 2000) and that it *increases* with the informativeness of stock prices (i.e. how much information stock prices contain about future earnings) (Durnev *et al.*, 2003). Together, these elements give empirical relations in finance a distinct character compared with the natural sciences but also compared with other social sciences.

1.4 Contents

In the eleven chapters that follow this introduction, the book discusses selected financial topics in their approximate order of historical development. This is a natural way to acquaint students with the evolution of financial–economic thought and it ensures increasing sophistication of the analyses. The second chapter provides a basis for the later investigations by briefly recapitulating some fundamental concepts from financial calculus, accounting and micro economics. Most students will have met these concepts before, but the summary operationalizes their knowledge to the level necessary in later chapters. Chapter 2 also contains a short description of how financial markets work in practice. Modern portfolio theory is the subject of Chapter 3. It starts with Markowitz's classic portfolio selection model and continues with the equilibrium pricing models that are its natural descendants, the Capital Asset Pricing Model and Arbitrage Pricing Theory.

Chapter 4 covers market efficiency, a concept that is counter-intuitive to many students with a background in the natural sciences. A thorough discussion of the concept and the presentation of extensive empirical evidence aim to overcome their scepticism. The fifth chapter presents Modigliani and Miller's ground-breaking analyses of the capital structure and dividend decisions, along with Myers' pecking order theory and recent empirical tests. The main insights from capital structure theory are applied, in Chapter 6, to the problem of valuing projects that are partly financed with debt.

From Chapter 7 onwards, option pricing pervades the analyses. The characteristics of options as securities are described in Chapter 7, which also lays the foundations for option pricing in state-preference theory. The binomial option pricing model completes the chapter's analyses in discrete time. Chapter 8 surveys option pricing in continuous time. It introduces the technique of changing probability measure with an example from gambling and proceeds with an informal derivation of the celebrated Black and Scholes formula, followed by a discussion of some of its properties and applications. Option-pricing techniques are applied to a variety of real options in Chapter 9 and three other problems in corporate finance in Chapter 10.

Hedging financial risks is explored in Chapter 11, along with the pricing of the main derivative securities involved in the process. Hedging techniques are applied to cross hedging in commodity markets and foreign exchange rate risk. The final chapter examines two more general problems in corporate finance, the agency relations that exist between the firm and its stakeholders, and corporate governance, the way in which firms are directed and controlled.

2 Fundamental concepts and techniques

This chapter summarizes the basic concepts and techniques that are used throughout the other chapters. We first look at the time value of money and common interest rate calculations. We then recapitulate how a firm's accounting system records and reports financial data about the firm. An example illustrates how these techniques and data can be used for investment decisions. Subsequently, we introduce the economic concepts of utility and risk aversion, and their use in financial decision making. The chapter concludes with a brief look at the role of financial markets, both from a theoretical and practical perspective.

2.1 The time value of money

2.1.1 Sources of time value

The time value of money can be summarized in the simple statement that €1 now has a higher value than €1 later. The time value of money springs from two sources: time preference and productive investment opportunities. Time preference, or 'human impatience' as the economist Fisher (1930) calls it, is the preference for present rather than future consumption. This is more than just impatience. Some consumption cannot be postponed for very long, for example the necessities of life. For other goods, the time pattern of people's consumptive needs is almost inversely related to the time pattern of their incomes. People want to buy houses when they are young and starting families, but if they had to accumulate the necessary money by saving, only a few could afford to buy a house before retirement age. Moreover, postponing consumption involves risk. Even if the future money is certain, the beneficiary, or the consumptive opportunity, may no longer be around. As a result, people require a compensation for postponing consumption and are willing to pay a premium to advance it.

The alternative to consumption is using money for productive investments. Productive means that the investment generates more than the original amount. This is found in its simplest form in agriculture where grain and livestock can be consumed directly or cultivated to give a larger harvest after some time. But the same principle applies to investments in machinery, infrastructure or human capital: by giving up consumption today we can increase consumption later.

The time value of money is expressed in a positive risk-free interest rate.[1] In free markets, this rate is set by supply and demand which, in turn, are determined by factors such as the amounts of money people and businesses hold, the availability of productive

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[1] Real-life interest rates often contain other elements as well, such as compensation for risk and inflation. We will deal with risk-adjusted rates later and assume no inflation.